

# Archive of Problems for High School Mathematics

A COLLECTION OF PROBLEMS FOR MATHEMATICS AT THE HIGH SCHOOL LEVEL

by

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## Preface

This archive comprises of a collection of problems in mathematics aimed at the high school level. Questions may be from various sources, some of which I can no longer identify for they were handed to me a long time ago as a print sheet.

I shall also note that the high school level of which I speak of is for the Malaysian national education system. Readers from other nations should not expect the level of this document to match the level of education in their own.

I am creating this archive with the purposes of collecting notable questions that I believe are great questions for a learner, that they are able to open their eyes to the topics that they are expected to learn in their time.

### *For Learners*

Marks are provided for each problem. Use them as a guide to suggest how much work is expected to be put in, in the sense where if you've ended up producing 3 A4/letter papers of math to answer a problem worth only 4 marks, you're likely thinking of the problem wrong, and you should go back to the drawing board.

The final answer for each problem, wherever sensible, will also be provided. You've probably been told a million times in your life<sup>1</sup> that the final answer doesn't matter, but what does is how you get there. Use the final answer to check for the accuracy of your work.

If you do not get the correct final answer, chances are you've did something wrong in your steps. Trace back, figure out what went

<sup>1</sup> And if you have not, you're hearing it here now.

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wrong, whether it was a silly mistake or a misconception, and rectify that. That's how you can learn mathematics, and virtually any topic in any field with a clear structure.

# 1 Inequalities

(1999, likely their SPM Trial)

1.(a) Find the range of  $x$  such that  $9 + 2x > 3$  and  $19 > 3x + 4$ . [3m]

(b) Given that  $2x + 3y = 6$ , find the range of  $x$  when  $y < 5$ . [2m]

(Answer) (a)  $-3 < x < 5$  (b)  $x > \frac{-9}{2}$

2. Find the range of values of  $x$  if  $(x - 2)(2x + 3) > (x - 2)(x + 2)$ . [3m]

(Answer)  $x < -1, x > 2$



## 2 Logarithms

(1997, likely their SPM Trial)

1. Show that  $\log_3 xy = 2 \log_9 x + 2 \log_9 y$ . Hence find the value of  $x$  and of  $y$  which satisfy the equation  $\log_3 xy = 10$  and  $\frac{\log_9 x}{\log_9 y} = \frac{3}{2}$ . [6m]

(Answer)  $y = 81, x = 729$

- 2.(a) Without using a calculator, find the values of  $3^{\log_3 7}$ . [2m]

- (b) Solve the equation  $5 \log_x 3 + 2 \log_x 2 - \log_x 324 = 4$  and give your answer correct to four significant figures. [4m]

(Answer) (a) 7 (b) 1.316

- 3.(a) Given that  $2 \log_3(x + y) = 2 + \log_3 x + \log_3 y$ , show that  $x^2 + y^2 = 7xy$ . [3m]

- (b) Without using a calculator, solve the equation

$$\log_9[\log_3(4x - 5)] = \log_4 2.$$

[3m]

- (c) After  $n$  years, the value of a new car is RM 60000  $\left(\frac{7}{8}\right)^n$ . Calculate the number of years it takes for the value of the car to be less than RM 20000 for the first time. [4m]

(Answer) (b) 8 (c) 9

(1998, likely their SPM Trial)

1. Given that  $\log_{\sqrt{x}} 4 = u$  and  $\log_y 2 = w$ , express  $\log_4 x^3 y$  in terms of  $u$  and/or  $w$ . [4m]

(Answer)  $\frac{6}{u} + \frac{1}{2w}$

- 2.(a) Given that  $\log_a 3 = x$  and  $\log_a 5 = y$ , express

$$\log_a \left( \frac{45}{a} \right)$$

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in terms of  $x$  and  $y$ . [2m]

(b) Find the value of  $\log_4 8 + \log_r \sqrt{r}$ . [3m]

(c) Find the value of  $x$  and of  $y$  that satisfy the simultaneous equations  $3(9^x) = 27^y$  and  $\log_2 y = 2 + \log_2(x - 2)$ . [5m]

(Answer)

(a)  $2x + y - 3$

(b) 2

(c)  $x = \frac{5}{2}, y = 2$ .

(1999, likely their SPM Trail)

1. Given that  $\log_2 3 = 1.585$  and  $\log_2 5 = 2.322$ , without using a calculator, find

(a)  $\log_2 45$  [3m]

(b)  $\log_4 \left(\frac{9}{5}\right)$  [3m]

(Answer) (a) 5.492 (b) 0.424

2.(a) If  $x = \log_2 3$ , find the value of  $4^x$ . Then, find the value of  $4^y$  if  $y = 1 + x$ . [5m]

(b) Given that  $\log_a 3 = 0.7924$ ,

i. prove that  $\log_a 27a = 3.3772$ , and

ii. solve the equation

$$\sqrt{3} \times a^{n-1} = 3$$

for  $n$ .

[5m]

(Answer) (a) 36 (b) ii.  $n = 1.3962$

(2000, likely their SPM Trial)

1.(a) Solve  $3^{\log_2 x} = 81$  for  $x$ . [3m]

(b) If  $3^{2x} = 8(2^{3x})$ , prove that  $x \log_a \frac{9}{8} = \log_a 8$ . [3m]

(Answer) (a)  $x = 16$

2.(a) Simplify  $\frac{\log_{12} 49 \times \log_{64} 12}{\log_{16} 7}$ . [3m]

(b) If  $3 \log_{10} xy^2 = 4 + 2 \log_{10} y - \log_{10} x$ , and  $x$  and  $y$  are positive integers, show that  $xy = 10$ . [3m]

(c) The total amount of money deposited in a cooperative after a period of  $n$  years is given by  $\text{RM } 2000(1 + 0.07)^n$ . Calculate the minimum number of year needed for the money to exceed  $\text{RM } 4000$ . [3m]

(Answer)

(a)  $\frac{4}{3}$

(c)  $n = 11$

(2001, likely their SPM Trail)



1. Given that  $\log_2 k = p$  and  $\log_3 k = r$ , find  $\log_k 18$  in terms of  $p$  and  $r$ .  
[4m]

(Answer)  $\frac{r+2p}{pr}$

- 2.(a) If  $\log_{10} x = 2$  and  $\log_{10} y = -1$ , show that  $xy - 100y^2 = 9$ . [3m]

(b) Solve

i.  $3^{x+2} = 24 + 3^x$ , and

ii.  $\log_3 x = \log_9(5x + 6)$ .

[3m]

(Answer)

(b) i.  $x = 1$

ii.  $x = 6$

(2002, likely their SPM Trail)

- 1.(a) Given  $\log_5 3 = k$ , and if  $5^{2\lambda-1} = 15$ , find  $\lambda$  in terms of  $k$ .<sup>1</sup> [3m]

<sup>1</sup>  $\lambda$  is pronounced like lambda.

(b) Solve

$$\log_2(7t - 2) - \log_2 2t = -1$$

for  $t$ . [3m]

(Answer) (a)  $\lambda = \frac{k+2}{2}$  (b)  $t = \frac{1}{3}$

- 2.(a) Given  $2 \log_4 x - 4 \log_{16} y = 3$ , express  $x$  in terms of  $y$ . [5m]

(Answer)  $x = 8y$

(b) Solve the simultaneous equations

$$2^{m-1} \times 32^{k+2} = 16 \quad \text{and} \quad 5^{-3m} \times 125^{3-k} = 1$$

for  $k$  and  $m$ . [5m]

(Answer)  $k = -2, m = 5$